

1 Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} - 8\frac{dx}{dt} - 9x = 9e^{8t}. \quad [6]$$

[illegible]

2 Let $I_n = \int_0^1 (1+3x)^n e^{-3x} dx$, where n is an integer.

(a) Show that $3I_n = 1 - 4^n e^{-3} + 3nI_{n-1}$. [3]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find the exact value of I_2 . [3]

[illegible]

3 The matrix $\mathbf{A}$ is given by

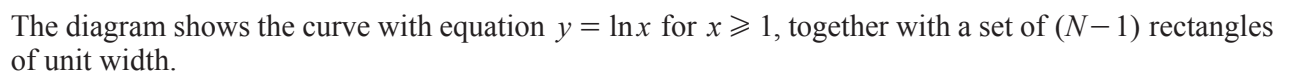
$$\mathbf{A} = \begin{pmatrix} 5 & -1 & 7 \\ 0 & 6 & 0 \\ 7 & 7 & 5 \end{pmatrix}.$$

(a) Find the eigenvalues of $\mathbf{A}$.

[4]

This image shows a full page of a worksheet designed for handwriting practice. It consists of multiple rows of horizontal dotted lines spaced evenly across the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

[Turn over


$$\ln N! > N \ln N - N + 1. \quad [5]$$
[illegible]

- (b)** Use a similar method to find, in terms of N , an upper bound for $\ln N!$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

5 The curve C has parametric equations

$$x = \frac{1}{2}t^2 - \ln t, \quad y = 2t + 1, \quad \text{for } \frac{1}{2} \leq t \leq 2.$$

(a) Find the exact length of C .

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings on the paper.

(b) Find $\frac{d^2y}{dx^2}$ in terms of t , simplifying your answer.

[4]

[illegible]

- 6 (a)** Starting from the definitions of $\tanh$ and sech in terms of exponentials, prove that

$$1 - \tanh^2 \theta = \operatorname{sech}^2 \theta. \quad [3]$$

The variables x and y are such that $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$, for $-\frac{1}{4}\pi < x < \frac{3}{4}\pi$.

- (b)** By differentiating the equation $\tanh y = \cos\left(x + \frac{1}{4}\pi\right)$ with respect to x , show that

$$\frac{dy}{dx} = -\operatorname{cosec}\left(x + \frac{1}{4}\pi\right). \quad [4]$$

- (c) Hence find the first three terms in the Maclaurin's series for $\tanh^{-1}\left(\cos\left(x+\frac{1}{4}\pi\right)\right)$ in the form $\frac{1}{2}\ln a + bx + cx^2$, giving the exact values of the constants a , b and c . [5]

[illegible]

7 (a) Show that an appropriate integrating factor for

$$(x^2 + 1) \frac{dy}{dx} + y\sqrt{x^2 + 1} = x^2 - x\sqrt{x^2 + 1}$$

is $x + \sqrt{x^2 + 1}$.

[4]

[illegible]

(b) Hence find the solution of the differential equation

$$(x^2 + 1) \frac{dy}{dx} + y\sqrt{x^2 + 1} = x^2 - x\sqrt{x^2 + 1}$$

for which $y = \ln 2$ when $x = 0$. Give your answer in the form $y = f(x)$. [7]

[illegible]

- 8 (a)** Use de Moivre's theorem to show that $\sin^6 \theta = -\frac{1}{32}(\cos 6\theta - 6 \cos 4\theta + 15 \cos 2\theta - 10)$. [6]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

It is given that $\cos^6 \theta = \frac{1}{32}(\cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10)$.

- (b)** Find the exact value of $\int_0^{\frac{1}{3}\pi} \left(\cos^6\left(\frac{1}{4}x\right) + \sin^6\left(\frac{1}{4}x\right) \right) dx$. [4]

[illegible]

- (c) Express each root of the equation $16c^6 + 16(1 - c^2)^3 - 13 = 0$ in the form $\cos k\pi$, where k is a rational number. [5]

[illegible]

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